Friday, April 27, 2018
6:13 AM

So we have established 3 notions on <u>entropy</u>:

a) Thermodynamic $dE = TdS$ w/ S non-decreasing

b) Statistical $S = -\sum p_i \ln p_i$ for a probability distribution $\{p_i\}$ w/ $\sum p_i = 1$

c) Von Neumann $S = -Tr\rho \ln\rho$ for any positive definite Hermitian operator ρ.

Is there overlap? Yes. If the probability distribution $\{p_i\}$ in (b) is appropriate, e.g. $e^{-\beta E_i}$ ($\beta \to \frac{1}{KT}$)
then the statistical entropy reproduces the thermodynamic entropy.

If ρ in (c) is a diagonalized density matrix $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$ then the Von Neumann entropy reduces to the statistical definition.

Then of course if $\rho = \sum e^{-\beta E_i} |E_i\rangle\langle E_i|$ then $(c) \to (a)$.

Technically, to connect (b) and (c) to (a) we can maximize S subject to a fixed average energy.

So does this mean that thermodynamic entropy is just the Von Neumann etropy of the underlying quantum system? After all, the world is fundamentally QM!
Well, no. The issue is that thermodynamic entropy arises because we choose to describe a system with a small number of parameters. We can choose more or less, but this will not impact ρ for our system.
So how is this freedom related to the QM picture? When we specify ρ we are using a Hilbert space which is presumably constructed in terms of observables. A full specification of ρ requires we use a complete set of compatible observables. But we could choose to use less, i.e. we can coarse grain the Hilbert space (use less than a complete set).

The take-away is that $S_{thermo} \geq S_{VN}$ (which will be important later on!).

Friday, April 27, 2018
7:57 AM

Last time I misspoke. Let me set the record straight.

Consider a probability distribution in (b) for a composite system w/ 2 variables $p_{AB}(a_j, b_k)$.
If this factorizes, i.e. $p_{AB}(a_j, b_k) = p_A(a_j)\, p_B(b_k)$ then $S(A) + S(B) = S(AB)$.
If not, then generally $S(A) + S(B) \geq S(AB)$ and we have "classical entanglement" (bag of heads).

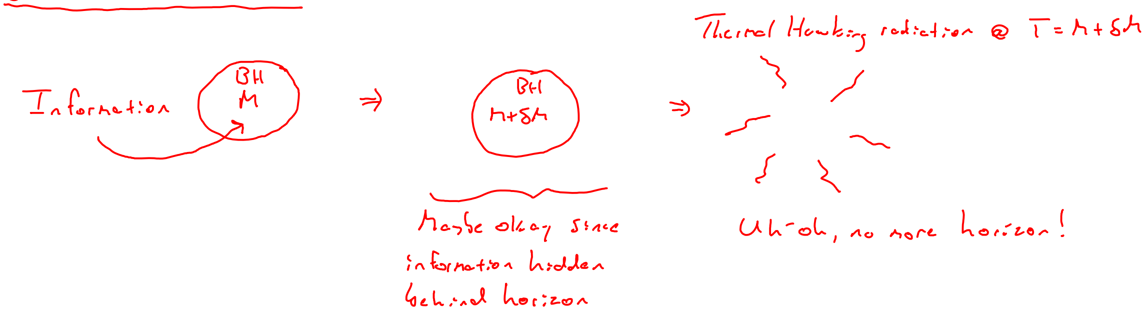
However what is true for (b) is that $S(AB) \geq \max(S(A), S(B))$, i.e. $S(AB)$ is greater than the larger of $S(A)$ or $S(B)$ even for classically entangled systems!

This is not true for (c), since as we discussed, we can take AB to be pure for which $S(AB) = 0$, but for "quantum entanglement" $S(A)$ and $S(B)$ are $\neq 0$. In fact in this case $S(A) = S(B) \neq 0$.

This being said, in classical applications of (b) to (a) we do find factoizable distributions so Zach is not a liar!

Friday, April 20, 2018
9:03 AM

Back to Black Holes



This was the original form of the BHI paradox, but it poses some issues.

1) Since it requires <u>complete</u> evaporation, it likely gets into the realm of QG (as the horizon shrinks to very small size) and the effective field theory methods that Hawking used will likely break down before the end of the process (they work fine as long as it is big).

2) You could imagine resolving the paradox by claiming a "remnant" that forms when QG becomes important, which contains the "lost" information.

However, by applying some of the more general ideas of quantum information and entropy, the paradox has been recently refined and shown to be more problematic. In fact we see problems well before complete evaporation!

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7:33 AM

Suppose we have a collection of information (bits) and an otherwise isolated BH. We know that the BH is radiating w/ temp. T and hence losing mass/energy and it's horizon size (hence entropy) is decreasing.

Now let's start feeding it bits of information at a rate s.t. the loss due to Hawking radiation is balanced by the gain from the bits.

The problem here is that we are putting information into the BH (which is lost to us), but the horizon size/entropy of the BH is remaining constant! Note, the BH remains large!

Uh-oh...

Friday, April 27, 2018
7:43 AM

Consider a pure quantum state that has collapsed into an isolated BH w/ energy $\underbrace{Mc^2}$.

determines horizon area, hence thermal entropy of BH

Now once we have a black hole we only have M, Q, J so we are forced into coarse-graining our description which leads to $S_{BH} = A$.

However since we started w/ a pure state $S_{VN} = 0$ for the black hole.

Now let it radiate. As it loses energy the area shrinks so $S_{BH} \propto Mc^2 \to 0$.
On the other hand, if we consider the entropy of the emitted radiation, this obviously starts at 0 and grows. Upon complete evaporation we should have $S_{rad} \propto 0 \to Mc^2$.

However we have full access to the emitted radiation, so no coarse-graining, i.e. $S_{rad} = S_{VN}$.

But if we consider the radiation that has left the BH (E) and the matter that is still inside (I) then we have $S_{VN}(EI) = \underset{\substack{\uparrow \\ \text{since we collapsed} \\ \text{a pure state}}}{0} \Rightarrow S_{VN}(E) = S_{VN}(I)$.

Now recall that for the S_{VN} of the BH we must have $S_{thermo} \geq S_{VN}$, but means that $S_{VN}(I) \to 0$. So as the pure state evaporates, $S(E) = S(I) \to 0$ which contradicts the result above!

In fact this poses a problem after only half of the BH has evaporated!